The Tale of One Variety (And Its Family)

Alexander Razborov, University of Chicago/Steklov Mathematical Institute

Fix an extremely large combinatorial structure: a graph, a hypergraph, a tournament etc. What can be said about frequencies/densities/probabilities with which small template structures of the same kind may occur in it? This question turns out to be of considerable significance both in pure mathematics (notably in Extremal Combinatorics) and various applications including network theory and statistical physics.

It turns out that the complete answer is provided by a ``simple'' countably-dimensional semi-algebraic variety that depends on the context. In the first part of the talk we will describe these varieties in sufficiently many details, highlighting their several geometric characterizations and other useful structure. In the second part we (partially) summarize our state of knowledge in these areas, both in terms of structural properties and concrete results and computations, and formulate several open problems.